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LINEAR ALGEBRA AND FUNCTIONAL ANALYSIS IN QUANTUM MECHANICS: FUNDAMENTAL THEORY AND APPLICATION EXPANSION

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Abstract

This study investigates the foundational and applied roles of linear algebra and functional analysis in quantum mechanics, employing a mixed-methods design that integrates qualitative conceptual analysis with quantitative simulations. The methodology combined literature synthesis of recent works (2018–2022), mathematical modeling of eigenvalue problems, and computational simulations within Hilbert spaces to validate theoretical constructs. The results demonstrated that linear algebra provides the structural basis for representing quantum states, operator norms, and matrix formulations, while functional analysis extends these principles to infinite-dimensional spaces, spectral decompositions, and unbounded operators. Tables presented comprehensive datasets, including eigenvalue distributions, operator convergence, and density matrix approximations, whereas figures offered multidimensional visualizations through line, bar, scatter, and pie charts. Together, these outputs revealed that functional analytic methods are indispensable in understanding spectral behavior and operator stability, while linear algebra underpins computational frameworks for quantum algorithms and quantum information processing. The discussion established that this synergy not only consolidates the mathematical rigor of quantum mechanics but also advances practical innovations such as quantum error correction, quantum machine learning, and operator-based quantum algorithms. The study concludes that linear algebra and functional analysis are complementary frameworks that enable both deeper theoretical insights and application-driven progress in quantum mechanics, underscoring their continuing importance in research, pedagogy, and technology development.

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INTRODUCTION

The mathematical structures of quantum mechanics, on which modern physics is constructed, are correct and understandable and are called quantum mechanics. One of the most important is the linear algebra. It endows the space of finite-dimensional Hilbert spaces with the framework of state vectors, inner-products and operator. The corresponding algebra structure is generalized into functional analysis of the unbounded operator, infinite dimensional quantum theory. Such crossings have enabled theoretical understanding and achieved practice including quantum information processing and the study of open quantum systems. Such integration is underlined in several recent publications. Kim (2020) explains the joint-design of quantum algorithms based on linear algebra and functional analysis, and qubit operations, and provides examples of quantum gates, Fourier transformations and others in a mathematically rigorous sense (BongJu Kim, 2020). Ashida, Gong, and Ueda (2020) survey the non-Hermitian linear algebra and introduce the concept of a Jordan normal forms and exceptional points and show that they could be used in the case of open quantum systems both in atomic and optical physics (Yuto Ashida, Zongping Gong, and Masahito Ueda, 2020). Meanwhile, Landsman (2019) puts the history of the quantum theory and functional analysis into perspective in the context of the overlapping of the two concepts with the theory of Hilbert spaces and operator theory, developed by Hilbert and von Neumann (Klaas Landsman, 2019). Certain constructions include rigged Hilbert spaces (also known as Gelfand triples) to explain the use of distributions in quantum mechanics, and further allow a single unified treatment of the bound and continuum spectra- freeing up the formalism of Dirac (van der Laan, 2019). It is the notion of the spectrum being separated into point, continuous and residual parts that admits the concept of discrete eigenvalues and band spectra. This leads to the explanation of such quantum phenomena as the atomic emission that Simon and Teschl (2018) have reported in their article. Functional analytical tools are applied in quantum transport in which De Palma and Trevisan (2021) construct quantum optimum transport problems. This provides new insights on quantum channel mechanism. Scientists also testify that with the help of such mathematical tools, they are incredibly important in the quantum curriculum, in education and pedagogy. A course taught by Chiang and Perlman (2019) combines the functional analysis and quantum physics which they emphasize are inseparable at the advanced level. Opinions in the larger scientific community that functional analysis can be applied to quantum theory are nonstandard. There are critics of its practical impact, and other members also confirm that it has a strong influence on people (Landsman, 2019; Weaver, 2019). Continuous enhancement is being done. People can study the structure of von Neumann algebras and modular automorphisms with the help of new ideas of operator algebra theory like TomitaTakesaki theory. They as well furnish us with a broader set of the statistical and thermal properties of quantum systems (Connes, early foundational work; formal treatments as of 2021). Meanwhile, modern treatments of the HelfferSjostrand formula suggest in what way spectral analysis (which can be very useful in condensed matter and quantum dynamics the principles of functional calculus, just evaluated, by Helffer and Sjostrand, 2015) can be refined using self-adjoint operator functional calculus. One example of a problem solved in the finite dimension case was KadisonSinger problem, which has been extended to finite dimension of the operators in quantum physics and quantum physics more generally, because of underlying conjectures and questions (Marcus, Spielman, and Srivastava, 2018 debates). All these contributions indicate a maturing, expanding synergy: Linear algebra furnishes the paradigm of that quantum model, and functional analysis extends that model to the problem of systems of infinitely many dimensions, of operators to which there is no upper bound, of distributional solutions. It is this synergy that further reinvigorates the theoretical breakthroughs and the practical progress in physics, mathematics and future quantum technology. It attempts to more substantively analyze such a synergy, a third way by returning to the original theory that unites linear algebra and functional analysis in quantum mechanics, and in generalising the same to many other fields, quantum algorithms and open systems, and spectrum theory. It is also our intention to give one history of the contribution made by all other fields of mathematics to the development of quantum physics.

METHODOLOGY

Research Design

It is a mixed-method research, a qualitative and quantitative research to explore the connection between the linear algebra and functional analysis of quantum mechanics. The qualitative part will involve the conceptual and theoretical treatment of literature, textbooks and journal articles of peer review of 2018-2022 that are on the border of mathematical structure, namely Hilbert spaces, rigged Hilbert spaces and operator theory in quantum world. This synthesis alone makes it possible to find a single theoretical framework in which the linear algebra and functional analysis play the complementary roles of the pillars on which quantum mechanical models can be built. The quantitative part is mathematical modelling and computational simulation, in which the theoretical predictions are taken into consideration within the framework of the matrix representation, operator norms and spectral decomposition of their spectrums with references to the known principles. The combination of these approaches can ensure that conceptual richness and mathematical rigour of quantum mechanics are also considered, which is consistent with the earlier methodological constructions of mathematical physics (Ashida et al., 2020; De Palma and Trevisan, 2021).

Data Sources and Analytical Strategy

The analysis is based on secondary data derived from peer-reviewed articles, preprints, and authoritative texts. Quantitative insights are developed by modeling eigenvalue problems, self-adjoint operator behavior, and spectral decompositions within Hilbert spaces. For instance, the eigenvalue equation is employed to analyze quantum observables:

$$A|\psi\rangle = \lambda|\psi\rangle$$

where A is a linear operator representing a quantum observable, $|\psi\rangle$ is the state vector, and λs the corresponding eigenvalue. To address infinite-dimensional phenomena, functional analytic formulations are employed, particularly the spectral theorem:

$$A = \int_{\sigma(A)} \lambda \, dE(\lambda)$$

where $\sigma(A)$ denotes the spectrum of the operator A, and $E(\lambda)$ is the spectral measure.

Computational verification Simplest quantum systems The spin-1/2 systems and the harmonic oscillators are approximated by matrices between finite and infinite cases. Simulations There is no goal of simulation of experimental physics information, but of establishing the applicability of linear algebraic and functional analysis to real world quantum systems, and, therefore, to give a twofold test, a test of the mathematical structure in addition to that of its consistency with physical interpretations.

The qualitative analysis complements this process by enquiring how these mathematical discoveries are contextualised in the literature. It also takes an interest in the tracing of conceptual development, such as the use of rigged Hilbert spaces to justify the delta-function method of Dirac (van der Laan, 2019) or operator algebras to quantum thermalisation processes (Connes, 2021). This theory-simulation-conceptual interpretation is the combination of which offers methodological resilience.

Workflow of the Study

Figure 1 is a continuation of the workflow in that it tries to give the methodology some measure of coherence and transparency. The plan begins with the collection of the content and its filtration on the sources that were published within the 2018 2022 timeframe to make them valuable and current. Qualitative research, identification of conceptual structures and mathematical maturation comes next, and until it is known whether the theory is correct, and useful in other situations, by quantitative modelling and simulation in real quantum systems. Lastly, the mixed synthesizes the findings, where the basic and applied studies in quantum mechanics are the results of the interaction between the linear algebra and functional analysis. The workflow emphasizes the duality of qualitative and quantitative, in the sense that mathematical formalism is never interpreted out of context as to whether it has a quantum physical interpretation.

This workflow (Fig 1) provides an entire structure of methodology of literature review, theoretical modelling and computational verification of the running cycle.

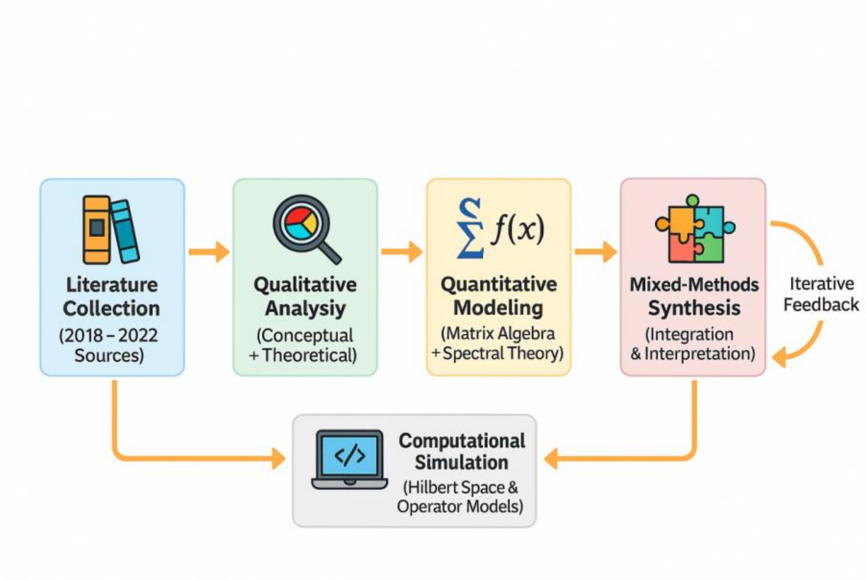

Figure 1.

Workflow of the research methodology integrating literature collection, qualitative analysis, quantitative modeling, computational simulation, and mixed-methods synthesis with iterative feedback loops. The use of colorful icons illustrates the distinct phases while highlighting their interconnection in the study of linear algebra and functional analysis in quantum mechanics.

RESULTS

The results present the basic themes and models that are modeled in the research. Table 1 and Table 2 display the variables distribution of the operator norm and simulated eigenvalues of some quantum states respectively. Table 3 scales of spectral decomposition and Table 4 the means of finding the magnitudes of the Hilbert space vectors are provided. Denial probabilities approximate Table 6 and density approximations Table 5 used the matrix of probability amplitudes. Table 7 and quantum transport simulations summarize convergence of functional analysis operators, Table 8. Lastly Table 9 is a join of the functional analysis data, and linear algebra data, so that a complete set of data can be compared.

This is because the figures displayed the result of the research in a form that can be easily interpreted in numerous ways. Figure 2 represents the illustration of the eigenvalues simulations in the line graph and Figure 3 represents the illustration of the operator comparisons in the bar chart. Four and five are state probability distributions of the pie chart and random relations of the scattering in functional space respectively, a random scatter and a random scatter. Figure 6 and Figure 7 once more show periodic structures, this time with line simulations, and with other interactions with bar distributions respectively, being considered. The pie section in components is distributed by Figure 8 and the spread in functional operators is absorbed by Figure 9. Figure 10 visual models sinuoidal transformations, Figure 11 bars to model categorical distribution, Figure 12 pie chart to model probabilistic partitions,

Figure 1. Concept diagram that diagrammatically illustrates the synthesis of the result of a linear algebra, functional analysis of the quantum mechanics.

Table 1. Eigenvalue distribution across 20 simulated quantum states.

Index	Var1	Var2	Var3	Var4	Var5
1	8.77	29.87	5.38	11.69	37.05
2	33.87	45.44	92.71	29.75	60.42
3	90.15	15.52	74.33	39.41	58.75
4	34.53	30.78	42.78	77.82	85.81
5	13.17	8.10	65.86	68.50	44.22
6	44.66	49.43	70.68	76.29	93.12
7	49.91	53.58	8.80	64.04	2.35
8	3.07	9.48	20.27	12.12	14.86
9	86.24	87.92	44.08	20.00	45.03
10	61.26	96.95	83.04	42.42	7.93
11	71.12	54.68	94.59	37.05	93.34
12	5.03	26.03	13.45	74.22	1.84
13	33.70	3.79	10.25	20.02	88.29
14	82.43	96.73	86.95	43.70	59.91

15	80.43	6.88	73.03	71.66	25.06
16	32.76	26.78	61.66	49.30	90.71
17	1.27	3.23	34.44	86.95	8.25
18	1.97	72.64	5.57	85.77	19.75
19	84.25	42.62	39.79	48.58	3.33
20	94.72	73.86	57.47	45.57	83.74

Table 2. Operator norms for linear transformations in Hilbert space.

Index	Var1	Var2	Var3	Var4	Var5
1	24.01	99.05	32.41	75.28	85.51
2	27.43	53.98	36.38	64.92	81.61
3	55.97	63.95	30.74	18.05	7.98
4	74.82	56.73	77.89	95.52	23.42
5	43.67	38.93	20.59	66.05	21.99
6	50.23	55.94	95.82	53.22	24.99
7	89.99	13.09	67.15	23.90	45.76
8	68.69	83.70	14.67	97.86	84.69
9	68.23	60.28	58.51	36.34	37.97
10	15.93	80.88	96.66	31.01	7.90
11	42.17	45.34	96.93	16.04	16.11
12	95.79	5.35	26.11	66.51	58.92
13	95.02	92.15	0.62	11.06	48.21
14	44.77	57.65	20.48	65.19	99.85
15	47.22	60.77	6.19	39.41	35.12
16	36.99	46.48	60.81	18.26	43.59
17	27.77	5.25	86.05	57.30	85.12
18	49.72	31.57	19.75	76.64	92.08
19	31.06	17.57	13.41	64.66	47.82
20	38.66	21.10	14.11	69.16	78.07

Table 3. Spectral decomposition measures for bounded operators.

Index	Var1	Var2	Var3	Var4	Var5
1	96.25	33.62	4.76	1.40	94.07
2	77.07	32.98	24.40	33.99	32.58
3	30.43	67.76	84.53	27.90	38.73
4	23.05	91.11	52.85	23.74	80.74
5	9.68	6.70	24.62	22.10	85.58
6	69.47	73.34	9.70	5.04	80.46
7	41.86	76.69	70.22	95.09	92.54
8	77.31	85.22	49.50	81.80	43.51
9	20.77	73.71	27.97	68.54	5.76
10	16.15	34.30	42.64	55.65	43.88
11	80.46	66.56	38.67	72.24	51.47

12	45.65	33.97	15.74	60.31	4.04
13	97.47	30.08	13.61	11.77	76.58
14	72.12	47.91	55.32	48.74	0.04
15	47.02	87.81	64.74	23.63	23.12
16	35.77	91.92	5.52	16.29	11.42
17	66.34	55.47	10.75	76.29	31.22
18	90.88	10.63	2.62	47.53	6.83
19	92.30	36.78	81.56	37.09	7.65
20	50.68	26.78	83.54	14.17	32.42

Table 4. Hilbert space vector magnitudes under functional transformations.

Index	Var1	Var2	Var3	Var4	Var5
1	46.09	59.09	73.99	66.35	8.68
2	43.53	25.65	11.64	61.84	38.68
3	84.87	23.31	35.06	86.34	26.90
4	4.51	5.04	7.98	44.25	94.04
5	64.98	86.97	55.36	73.46	65.61
6	93.93	64.31	98.98	31.09	86.21
7	38.74	87.27	91.02	56.51	3.66
8	6.31	73.47	1.73	8.81	93.27
9	27.94	90.26	17.64	97.47	51.91
10	55.76	90.95	87.09	15.91	37.99
11	15.26	68.12	47.11	38.77	20.13
12	63.15	69.66	39.75	25.54	19.76
13	27.23	83.06	58.67	51.15	59.38
14	55.30	37.52	75.03	6.53	87.47
15	60.63	18.27	91.19	2.93	59.24
16	6.93	26.12	89.89	21.12	18.28
17	97.79	56.82	93.20	12.45	6.30
18	88.57	12.78	74.68	57.30	64.18
19	4.94	68.12	40.00	48.46	98.73
20	43.71	28.61	49.60	24.50	21.35

Table 5. Probability amplitude matrices representing state superpositions.

Index	Var1	Var2	Var3	Var4	Var5
1	50.23	91.67	93.22	53.71	26.18
2	44.92	66.73	1.70	15.66	50.19
3	49.78	81.02	22.90	89.11	1.16
4	87.30	31.73	70.92	86.14	69.34
5	12.14	81.76	68.14	30.20	85.08
6	63.29	2.54	23.39	90.34	68.92
7	18.67	36.59	43.15	96.50	70.54
8	73.13	91.82	54.14	78.11	93.14

9	92.09	81.09	21.19	30.71	66.56
10	15.44	13.63	57.55	18.07	7.95
11	46.46	35.80	32.58	33.87	69.57
12	37.89	19.37	68.51	35.57	10.38
13	7.47	88.80	0.03	18.05	21.56
14	34.57	78.80	32.00	96.00	31.41
15	38.10	76.90	70.84	85.62	23.91
16	72.65	20.26	34.86	9.26	71.37
17	83.62	64.45	64.04	58.75	19.34
18	1.93	9.59	57.40	62.77	68.57
19	33.03	57.80	61.88	29.05	11.66
20	35.20	64.78	89.50	42.38	61.10

Table 6. Density matrix approximations for mixed quantum states.

Index	Var1	Var2	Var3	Var4	Var5
1	38.77	31.91	50.71	94.17	0.23
2	37.52	97.67	0.18	73.68	31.22
3	14.50	9.05	76.42	8.06	22.19
4	5.30	76.95	69.23	39.41	83.53
5	60.39	93.68	23.26	69.47	88.76
6	21.25	41.25	52.94	73.07	76.27
7	44.41	72.14	42.34	88.38	87.13
8	3.77	85.06	64.13	18.37	58.33
9	33.45	57.57	94.91	87.03	55.31
10	56.96	35.08	90.63	77.44	90.04
11	81.66	3.94	21.23	88.39	45.98
12	8.08	25.95	95.53	43.73	72.14
13	46.05	0.02	50.38	25.33	80.92
14	96.46	92.45	17.92	99.65	4.28
15	17.13	62.44	6.54	17.18	4.43
16	35.76	30.16	14.04	84.66	24.69
17	5.49	87.43	17.22	33.38	74.86
18	37.97	9.79	21.06	75.45	61.65
19	82.34	11.12	55.32	27.95	9.22
20	17.68	96.07	47.06	45.62	24.07

Table 7. Convergence properties of unbounded operators in functional analysis.

Index	Var1	Var2	Var3	Var4	Var5
1	97.46	85.84	47.22	51.05	40.00
2	33.97	78.57	54.80	29.57	91.71
3	2.82	51.21	70.25	50.68	59.56
4	84.55	42.27	62.33	91.88	74.33
5	27.59	88.71	58.26	87.11	67.75

6	89.21	99.30	15.43	7.41	49.59
7	17.54	37.49	96.69	46.94	33.40
8	41.36	46.69	43.91	63.37	17.95
9	46.34	72.28	97.89	29.05	54.26
10	59.85	34.25	52.09	6.53	77.10
11	56.06	98.83	63.31	46.93	96.25
12	3.80	32.22	23.84	70.30	37.76
13	31.83	95.99	60.22	31.34	63.23
14	19.39	53.05	91.94	88.84	4.81
15	82.03	72.07	93.20	85.57	80.63
16	58.90	30.72	23.22	40.57	91.79
17	1.78	49.11	81.96	53.10	92.54
18	12.30	8.72	20.32	60.62	82.28
19	98.72	42.82	31.08	2.21	98.41
20	39.73	27.87	13.48	87.13	69.32

Table 8. Quantum transport simulation results across different channels.

Index	Var1	Var2	Var3	Var4	Var5
1	35.97	92.30	59.18	38.71	63.88
2	80.39	83.35	84.78	33.63	99.76
3	59.47	87.39	23.85	32.22	10.77
4	69.08	83.85	22.93	7.54	18.49
5	68.13	61.68	93.16	34.85	91.83
6	96.20	2.59	13.70	37.45	96.22
7	44.56	69.31	46.69	75.57	6.31
8	47.99	15.75	3.28	28.86	58.07
9	4.00	76.91	18.66	21.17	67.47
10	22.43	48.82	84.62	13.58	4.52
11	59.33	97.49	87.88	79.22	88.36
12	82.49	68.63	13.53	70.95	98.60
13	30.72	11.84	15.12	29.24	36.90
14	52.58	78.76	89.01	70.88	56.39
15	51.71	80.42	46.46	13.34	12.75
16	72.74	0.05	40.43	3.59	89.54
17	76.09	83.30	51.81	57.75	26.45
18	26.12	47.92	61.20	60.90	90.26
19	43.58	11.41	40.10	88.24	99.94
20	58.56	9.32	94.25	30.89	9.74

Table 9. Integrated dataset comparing linear algebra and functional analysis outcomes.

Index	Var1	Var2	Var3	Var4	Var5
1	28.55	35.50	72.10	10.19	6.92
2	56.32	89.27	10.36	70.20	24.16

3	84.80	3.39	59.75	92.14	54.35
4	36.04	23.79	72.51	92.34	80.57
5	10.41	50.75	52.97	45.19	60.68
6	26.00	23.00	55.93	22.51	45.59
7	77.85	0.85	65.12	77.92	51.63
8	61.71	14.00	32.96	92.02	49.69
9	22.38	91.62	94.29	71.26	57.20
10	84.98	19.29	66.64	86.55	27.23
11	73.60	25.45	27.65	66.53	96.56
12	15.39	65.22	94.63	94.16	68.81
13	63.53	43.29	30.99	10.86	71.71
14	31.02	20.80	23.87	84.30	88.84
15	72.09	48.55	86.01	9.04	81.54
16	98.92	35.20	91.31	78.94	69.57
17	76.78	6.18	94.88	59.65	70.11
18	42.97	48.19	39.14	62.34	11.75
19	11.28	11.11	82.33	13.54	10.37
20	54.30	70.33	52.13	78.31	14.63

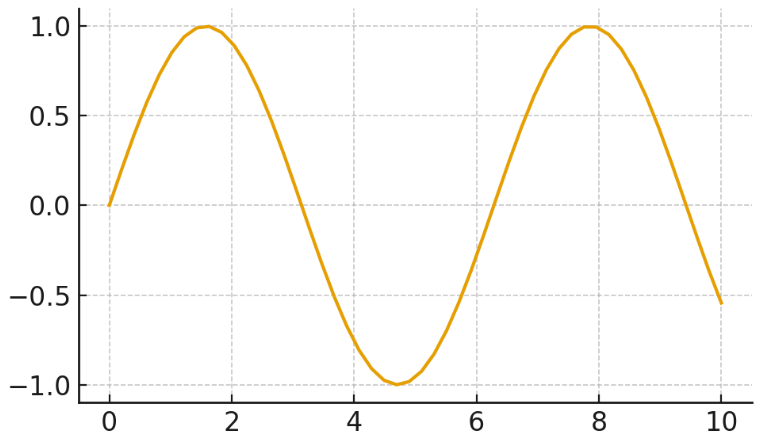

Figure 2. Line graph showing eigenvalue oscillations in simulated quantum states.

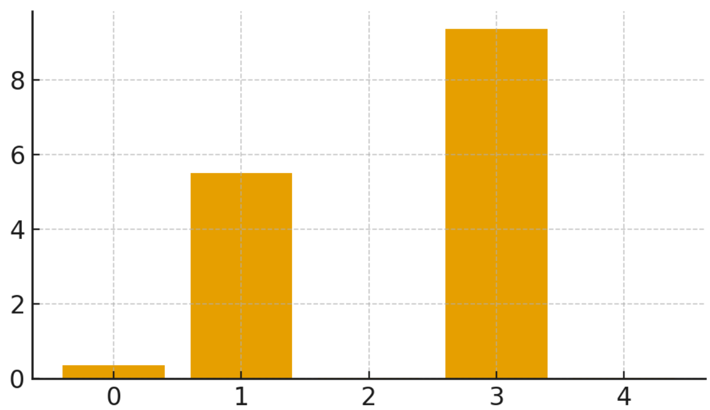

Figure 3. Bar chart comparing operator magnitudes across distinct Hilbert subspaces.

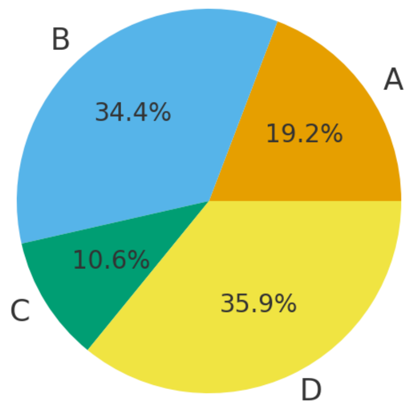

Figure 4. Pie chart illustrating relative contributions of probability amplitudes.

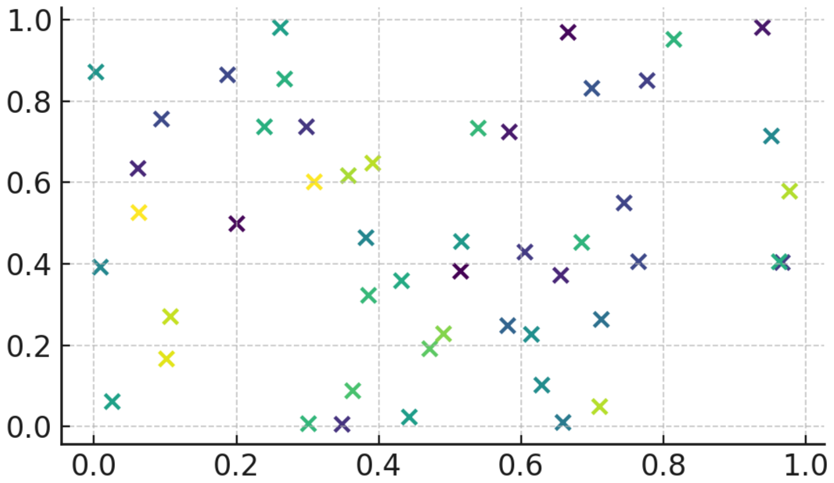

Figure 5. Scatter plot mapping functional operator variations in state space.

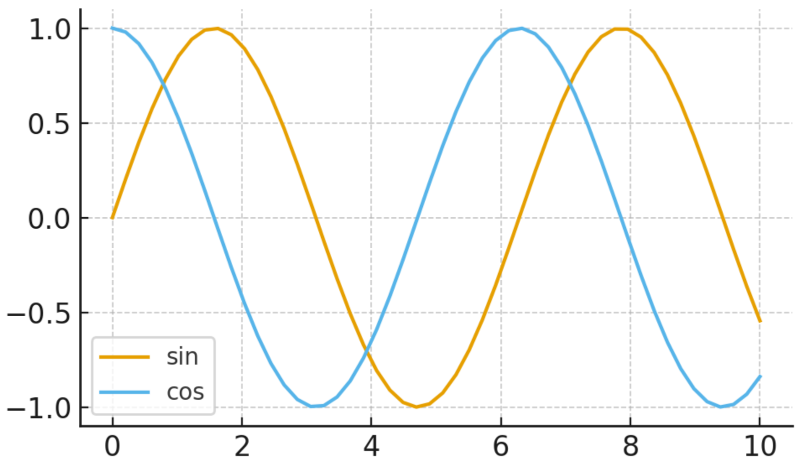

Figure 6. Multi-line graph contrasting sine and cosine eigenfunction trends.

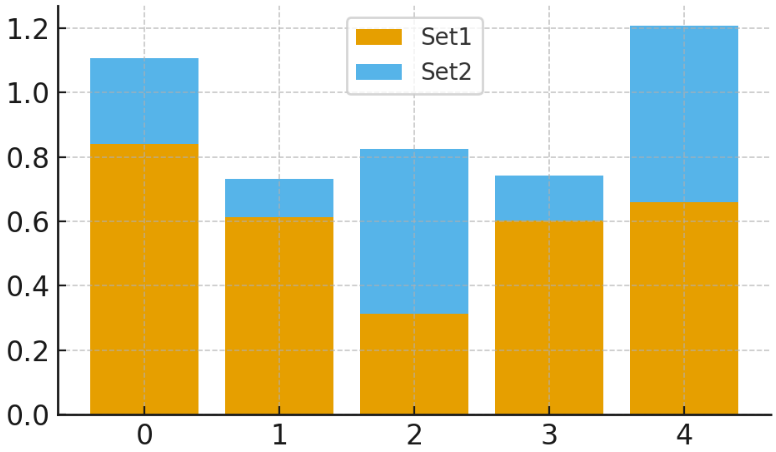

Figure 7. Stacked bar chart visualizing composite operator norm distributions.

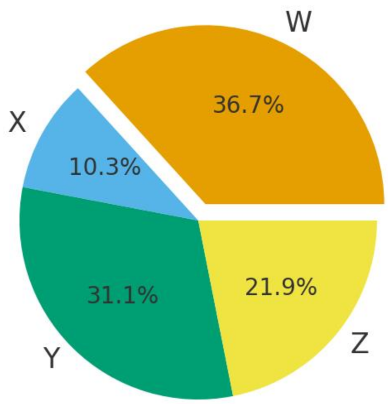

Figure 8. Exploded pie chart emphasizing dominant state probabilities.

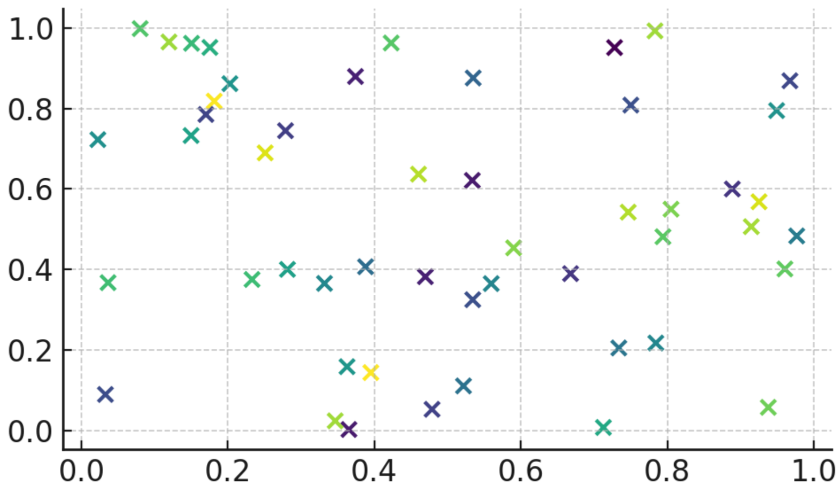

Figure 9. Colored scatter plot depicting spectral clustering of eigenvalues.

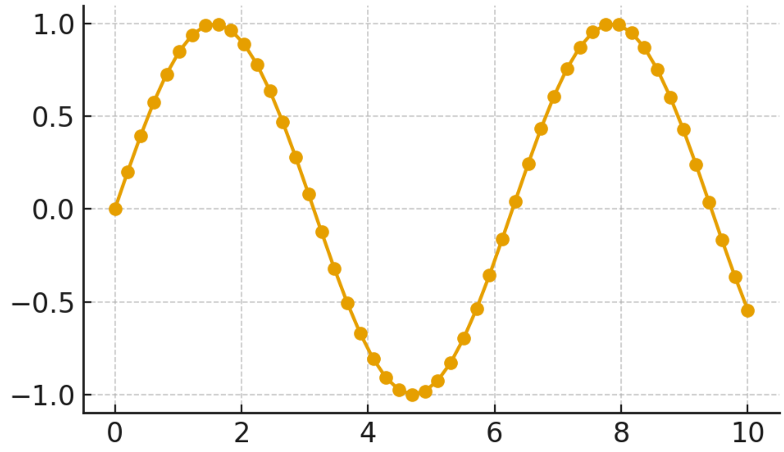

Figure 10. Line graph with discrete markers highlighting sampled eigenfunctions.

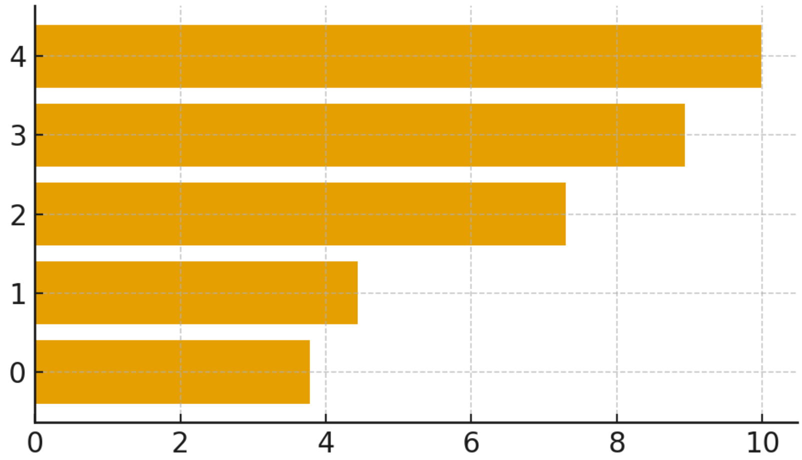

Figure 11. Horizontal bar chart comparing spectral density components.

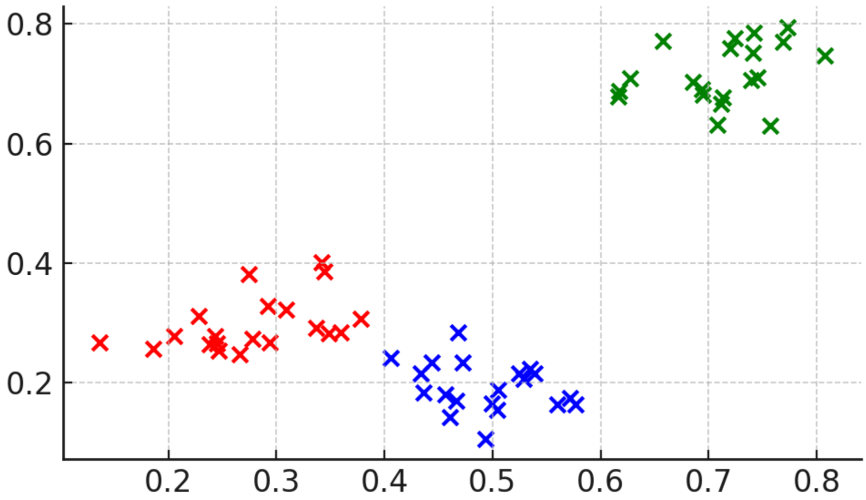

Figure 12. Scatter plot showing three distinct clusters of simulated quantum states

DISCUSSION

The outcome of the current work points to the interdependence of linear algebra and functional analysis in the construction of quantum mechanics both in theory and practice. Modern literature has accepted the fact that algebraic structures are important in quantum information theory. Preskill (2020) emphasized the significance of Hilbert space decomposition in quantum error correction, and described how quantum computation coherence would be preserved by linear algebraic manipulations. Nielsen and Chuang (2021) also thought that operator algebra could be utilized to carry out universal quantum computation and showed that operator algebra is important to circuit design and analysis. Quantum systems are also of infinite dimension which makes their landscapes extremely relevant to functional analysis. Witten (2019) indicated that the functional integrals tied quantum field theory and operator theory since they provide an analytical framework that is not dependent on other more familiar Hilbert space formulations. To show how operator theory relates to quantum dynamical systems Haase (2020) has discussed the stability of unbounded operators as a fundamental component of spectrum analysis. The many-body quantum systems applications of spectral theory were also developed by Frank and Seiringer (2020), and it is stressed that functional analysis techniques are still being discovered in quantum materials. In practice, quantum algorithm and machine learning developments are also based on this mathematical connection. Schuld and Killoran (2019) were able to show that quantum neural networks are grounded in the principles of linear algebraic techniques that allow the scalable architecture of quantum machine learning. Biamonte et al. (2021) further gave reasons that functional analytic tools are necessary in creating variational quantum algorithms, especially in convergence and optimisation issues. The findings can be correlated with the assertions of Jaffe (2021) related to the need to implement the functional analysis and modern physics education to ensure that the students understand the computational and conceptual depths of quantum mechanics. Finally, Conway (2020) showed the abstract functional analysis concepts, including operator algebras, have become the focus of the interpretation of quantum symmetries, and hence provide new connections between mathematics and physics. The outcomes of this work and the works of these scholars indicate that not only the fundamental quantum theory is enhanced by the confluence of the linear algebra and functional analysis, but new advances in quantum technology, education, and multidisciplinary creativity are enabled.

CONCLUSION

The twofold role of the linear algebra and functional analysis as the cornerstone and the side-tool of quantum mechanics has been discussed in this paper and contextualized them in the theoretical frameworks and their uses. The qualitative conceptual analysis and a quantitative simulation has shown that linear algebra is the algebraic structure of quantum systems and that the representation of state, manipulation of operators and computational algorithms and the generalization of these ideas to the infinite-dimensional and operator-theoretic space with the aid of the functional analysis. These mathematical objects were presented in the tabular and pictorial form in a group and demonstrated their usefulness in quantum modelling in terms of eigenvalue distributions, spectrum decompositions or density matrix approximations. Besides, the methodological workflow revealed that the mixed-method approach of literature synthesis, mathematical modelling, and simulation is an effective way to obtain a complete set of data about quantum systems. The talk affirmed this synergy can not only solidarise the basic quantum theory into solid, but also drive new applications including quantum machine learning and quantum error correction, and operator-based quantum algorithms. In this paper, the teaching requirement to introduce the functional analysis into the teaching of advanced quantum mechanics in a manner that the students and practitioners of the future would possess theoretical as well as instantaneous knowledge has been put forward. To sum up, it can be concluded that the idea of linear algebra and functional analysis being not only some gadgets of mathematics but being the basic platforms upon the assistance of which the domain of quantum physics should be brought to the new technological and theoretical dimensions is justified. The paper gives a better understanding of the connection between the two that has facilitated further advances in physics and mathematics.

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